Compact, holographic correction of aberrated telescopes

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Abstract
We demonstrate a compact reflector telescope design incorporating the holographic correction of a large, low quality primary spherical mirror using a laser beacon located at the center of curvature. The simple design makes use of conventional optics and is easily scalable to much larger apertures. Experimental results indicate diffraction limited performance from a heavily aberrated 0.5m diameter spherical mirror.

Keywords: Holography, Telescope, Aberration Correction

1. Introduction
In recent publications\textsuperscript{1-3} we have described and demonstrated practical concepts for using holographic optical elements to correct aberrations in large, low-quality telescope primaries, for applications ranging from space or moon based telescopes to ground based lidar. We have shown that a hologram recorded using a laser beacon located in the far field of a telescope can correct a primary element with hundreds of waves of aberrations to diffraction limited performance over a useful but limited bandwidth\textsuperscript{1,2}. More recently\textsuperscript{3} we have shown that an off-axis proximal beacon can result in a useful compact design with no obscuration of the primary. The latter approach suffered however, from a large amount of introduced astigmatism which was difficult to remove with conventional optical components. While computer generated holograms could be used to remove the astigmatism, we have chosen to concentrate on a simpler on-axis configuration suitable for the correction of large spherical surfaces. This approach is advantageous because it does not introduce any unnecessary additional aberrations, and can produce diffraction limited performance from an inexpensive, low quality primary using simple, readily available optical components.

The basic concept of holographic correction of a refractor telescope is reproduced from Ref. 1 in Fig. 1. An image hologram of the aberrated objective is recorded using a coherent light source (beacon) at infinity and a diffraction limited reference beam [Fig. 1(a)]. If the beacon is used to reconstruct this hologram, the original diffraction limited reference beam is recreated with the hologram effectively subtracting the aberrations on the incoming beam [Fig. 1(b)]. If instead the incident light comes from a distant object, the hologram will still subtract the aberrations but not the object information, resulting in a reconstructed beam from which a perfect, unaberrated image of the object can be produced [Fig. 1(c)]. In this paper, we will demonstrate for the first time, a compact adaptation of this concept which uses a spherical reflector and a beacon located at the center of curvature.
Figure 1: (a). Recording: A plane wave from a distant beacon is used to make a demagnified image hologram of the aberrated primary. (b). Reconstruction: The beacon light is used to reconstruct the original diffraction limited reference beam. (c). Imaging: Instead of the beacon, a distant object is viewed with the telescope. The additional object wave information is retained in the reconstructed reference beam, and an aberration-corrected image is formed.

2. The Proximal On-Axis Beacon
The On-Axis Design
The most compact configuration for recording an image hologram of a spherical mirror using a point-source beacon is shown in Fig. 2(a). The beacon is a diffraction limited spatial filter located at the center of curvature of the aberrated mirror. A high quality camera lens (L1) is used to collimate the reflected light and form a demagnified, flat-field image of the mirror on the hologram. A hologram is recorded with this object wave and a diffraction limited, plane wave reference beam.

In a telescope, light from a distant object is brought to a focus at half the radius of curvature of the mirror. In order to use the hologram as an optical element to correct the mirror when used as a telescope, the hologram must either be moved to the focal plane or, in the case of modest aberrations, an image relay system could be employed. Due to the heavily aberrated mirror used, we chose the former solution in this experiment. In Fig. 2(b), collimated light is focused by the aberrated mirror, then re-collimated by a second camera lens (L2) which also images the mirror on the plane of the hologram. The camera lenses must be chosen to give the same image magnification
on the hologram during recording and reconstruction and the hologram must be accurately relocated so that its recorded image is congruent to the image in the new location. It is also important to ensure that the divergence of the light incident on the hologram is the same during recording and reconstruction to avoid the introduction of additional geometrical aberrations in the corrected beam. This can only be accomplished by collimating both the object beams, since the image size must remain the same while the speed of the system has doubled.

![Diagram](image)

**Figure 2:** (a). Recording: The beacon illuminates the mirror from the centre of curvature. The reflected light is collimated and the mirror is imaged by the camera lens on the plate where a hologram is recorded. The off-axis angle has been exaggerated for clarity. (b). Reconstruction: Collimated light from the parabolic mirror is focussed by the aberrated mirror and collimated by the second camera lens, L2, to reconstruct the reference beam.

The reconstructing object beam will differ from the recording object beam in two ways: spherical aberration is not present during recording but is present on reconstruction, and the angular difference between the recording and reconstructing rays over the mirror surface will result in a difference in the perceived height of a mirror deformation. Both of these factors will affect the fidelity of the reconstructed reference beam, and will be discussed below.

**Mirror Aberrations**

When the recording and reconstructing rays differ in their wavelengths and angles of incidence, the phase change caused by the reflection from an aberration will differ, resulting in imperfect aberration correction. This effect has been calculated for a completely general geometry, but for the present case, with the beacon on-axis, it can be described using the simplified arrangement of Fig. 3. The aberration is assumed to be a bump of height \( h \) above the unaberrated surface. The
angle of incidence and wavelength of the recording ray are $\alpha$ and $\lambda$ respectively, and the corresponding quantities for the reconstructing ray incident on the same point are $\alpha'$ and $\lambda'$.

![Figure 3: A ray from the recording beacon is shown reflecting off a bump on the mirror surface (at A) at an angle $\alpha$ to the mirror normal (R). The phase difference between this ray and one which would have reflected off an unaberrated mirror (at B) is $\phi = (AB + BC)/2\pi\lambda = h \cdot \cos \alpha / \pi \lambda$. A reconstructing ray will make an angle $\alpha'$ to the normal at the same point A.](image)

Assuming perfect imaging properties of the secondary optics, it is easily shown that if the phase error due to the aberration recorded on the hologram is $\phi$, and if the phase error experienced by the reconstructing ray from a distant object is $\phi'$, then the error in the resulting phase correction is

$$\delta \phi = \phi - \phi' = \frac{h}{\pi} \left( \frac{\cos \alpha}{\lambda} - \frac{\cos \alpha'}{\lambda'} \right)$$  \hspace{1cm} (1)

If the aberrated telescope was used to observe a distant object without aberration correction, the phase error introduced would be $\phi'$. The factor $K$ by which the error on the reconstructing ray is reduced by the hologram is thus

$$K^{-1} = \frac{\delta \phi}{\phi'} = \left[ \frac{\lambda'}{\lambda} \frac{\cos \alpha}{\cos \alpha'} - 1 \right]$$  \hspace{1cm} (2)

In the case of a spherical mirror used to observe a point at infinity, and a laser beacon located at the radius of curvature of the spherical mirror, the expression for the correction factor can be simplified to be,

$$K^{-1} = \left[ \frac{\lambda'}{\lambda} \frac{x}{x^2 - 1} \right]^{1/2} - 1$$  \hspace{1cm} (3)
where $x=R/r$, $R$ is the radius of curvature of the mirror and $r$ is the distance from the center of the mirror to the point of incidence under consideration. Assuming an identical wavelength for recording and reconstruction, the correction factor is infinite at the center and decreases towards the rim of the mirror. This is shown in Fig. 4, where the minimum correction factor is shown as a function of mirror speed ($F=x/4$). As an example, the minimum correction factor at the edge of a spherical mirror with dimensions, $R=5.2\text{m}$ and $r=0.225\text{m}$, is $1067$ which means that an aberration will be reduced by at least this factor anywhere on the mirror.

![Figure 4: The minimum correction factor $K$ is plotted as a function of mirror speed, $F=x/4$. For example, with an F/5.8 mirror, the minimum correction (which is at the edge) will be 1067 as indicated. As we move towards the center of the mirror, the effective speed decreases and the correction factor will increase until it reaches an infinite value at the center.](image)

**Spherical Aberration**

With the beacon at the center of curvature of the spherical mirror, there will be no spherical aberration recorded. However, when reconstructing with a distant object, spherical aberration will be present. A successful telescope configuration must provide for the removal of this aberration. We have previously considered a variety of possible options for the removal of spherical aberration\(^3\), and choose here the simplest and most promising approach where the required amount of spherical aberration is included in the object beam during recording, resulting in a reconstructed object beam which is aberration free. The recording scheme used is shown in Fig. 5. The beacon light produced by a diffraction limited spatial filter is passed through a simple lens which introduces the spherical aberration and refocuses the light to form a new, aberrated beacon. The positions of the lens and the spatial filter aperture used are optimized with an optical ray tracing code to produce the spherical aberration required. From this point onwards the scheme is exactly as before.
Figure 5: Recording: The beacon light is spherically aberrated using a simple plano-concave lens. The focussed light then continues on to illuminate the mirror as before. The off-axis angle has been exaggerated for clarity.

3. Results

We used a 0.9m diameter, 5.2m radius of curvature spherical mirror, made by slumping 12mm thick plate glass. The wavefront error is up to 100 waves and the uncorrected focal spot is 8mm in diameter. In this experiment we tested only the central 0.45m diameter portion of the mirror, as this was the diameter of the diffraction limited parabolic mirror available as a collimator. An interferogram of this central portion is shown in Fig. 6(a).

The spherical aberration was added to the object beam using a plano-convex singlet (\(d=30\)mm, \(f=100\)mm) as described above. The optimized distance between the lens and the spatial filter was calculated to be 179.9mm, with a residual peak-to-valley error of \(\lambda/10\) over the entire aperture. This limit is due to the mismatch between the spherical aberration from a lens and a mirror. It was also calculated that a positioning error of \(\pm 1\)mm would increase the residual error by only \(\lambda/10\) and an error of \(\pm 5\)mm would give an increase of \(\lambda\). The mirror was imaged with a 100mm camera lens and the hologram was constructed with a diffraction limited, collimated reference beam. The bleached hologram was recorded on Agfa 10E75 plate film using a HeNe laser (\(\lambda=632.8\)nm) and was \(\sim 10\)mm in diameter. The typical diffraction efficiency (useful diffracted beam power/incident power) was 40%.

The hologram was reconstructed using a collimated beam produced by a spatial filter at the focus of a parabolic mirror (\(f=2m, d=450\)mm). The aberrated mirror was imaged with a high quality zoom lens, adjusted to give the correct image size at collimation. The reconstructed beam emerged from the circular hologram at an angle, and therefore had an elliptical cross-section. The aspect ratio was corrected, when necessary, by passing the beam through a prism or by simple digital image processing.

The reconstructed beam was compared interferometrically to the original reference beam used in recording. The results are shown in Fig. 6, and indicate a final wavefront error of \(\lambda/2\) peak-to-valley and 0.11\(\lambda\) rms. The calculated correction factor for this experiment has a minimum value of 1067 [Fig. 4] at the mirror edge. From the interferogram in Fig. 6(a), we can see that the maximum error of the mirror close to the edge is \(\sim 100\lambda\), implying that the maximum uncorrected surface error should be \(\sim \lambda/10\) at the edge.
Figure 6: (a). Before correction. The interference pattern shows the wavefront error over the 0.45m aperture. (b). After correction.

The residual wavefront error was limited by several factors. Firstly, errors introduced by non-common optical elements used in either recording or reconstruction, including the parabolic mirror, increased the error of the final wavefront. The parabolic mirror was tested by the manufacturers and known to have an error of $0.23\lambda$ peak-to-valley ($0.04\lambda$ rms) with an overall figure similar to that of our reconstructed wavefront. A further $0.03\lambda$ rms wavefront error was expected to remain due to the uncorrected spherical aberration.

The telescope was tested as an imaging instrument by placing a resolution chart at the focus of the parabolic collimator and illuminating it with diffused laser light. The results are shown in Fig. 7.

Figure 7: (a). USAF resolution chart before correction - showing Columns 2 and 3. (b). After correction showing the diffraction limited resolution with bars resolved to Column 7 Line 3.
Before correction, even Columns 2 and 3 are heavily blurred but after correction the image of the chart is resolved to Column 7 Line 3, which corresponds to diffraction limited performance for the mirror in this configuration. The field of view is difficult to assess due to the small field of view of the collimator itself, but the resolution chart does maintain a sharp image quality over the whole pattern.

As demonstrated previously\(^2\), the holographic concept has a bandwidth limited by the magnitude of the surface errors to be corrected. From Eqn. (3) we can predict the correction for different wavelengths. For a hologram written in the red \((\lambda=632.8\text{nm})\) and reconstructed in the green \((\lambda'=532\text{nm})\), the calculated correction factor is 6.3. Although this would not result in a high resolution telescope at this wavelength we present experimental data as further verification of the theory. We used a CW, doubled-YAG laser to reconstruct the hologram, giving the results shown in Fig. 8. The correction factor suggests that for every six fringes on the interference pattern of the uncorrected mirror [Fig. 6(a)] there should be one fringe on the corrected pattern at this wavelength [Fig. 8(a)]. This indeed seems to be the case. The resolution is indicated by the image in Fig. 8(b). Column 3 Row 6 (14.3 lines/mm) can be resolved in the vertical direction and Column 4 Row 3 (20.1 lines/mm) in the horizontal.

![Figure 8](image.png)

**Figure 8:** (a). The interference pattern for reconstruction at \(\lambda'=532\text{nm}\). (b). The resolution chart image at the same wavelength.

**Conclusion**

We have presented a compact design for the holographic correction of a spherical reflector telescope. The design included a method of incorporating spherical aberration correction during the recording of the hologram to cancel out that which is present when imaging a distant object. We have demonstrated the successful application of this scheme in correcting a large diameter, heavily aberrated telescope. Diffraction limited correction is implied, with the remaining error consistent with the limited quality of non-common optical components used. The correction scheme was purposefully designed using conventional optical components for ease of subsequent scaling. The experimental results do not indicate a limit to the size of the aperture which could be corrected by this method.
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References